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Investigation of Bubble Dynamics of a Second Grade Visco-elastic Fluid

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Abstract

Theoretical analysis will be made on the bubble dynamics of a second grade viscoelastic fluid. The flow will be considered to be laminar and incompressible. Theoretical results include the time wise variations in the bubble growth and mass transfer during growth. Some results will be obtained numerically using RK-4 method of bubble growth and mass transfer with effects of physical quantities, namely, viscosity, density, shear stress, pressure difference and viscoelastic parameter.

Keywords: Bubble growth, Second grade fluid, Shear Stress, Non-dimensionalization, Similarity transformation, Mass transport, RK-4 method.

1 Introduction

The investigation of a spherical bubble satisfying the constitutive equation of non-Newtonian fluids are of great importance with a broad range of practical application from sedimentation of muds and slurries to processing of filled polymer melts. Many researchers studied bubble dynamics with viscous liquids except few authors studied it with inelastic fluid such as Power law fluid model and viscoelastic fluid such as Oldroyd B. The kinetics of phase change plays important roles in many fields of science and technology. Phase changes proceed through the nucleation and growth of new phase. Studies on the nucleation and growth are necessary to understand various physical phenomena for example, bubble formation in ascending magmas and condensation of solid dusts in cooling gas. There have been a number of studies of the expansion of a spherical gas bubble due to diffusion of gas from the surrounding liquid.

The study of bubble dynamics was initially performed by Rayleigh [1] in 1917, who made analysis in large mass of liquid of the collapse of an empty cavity. In this paper, Rayleigh studied the gas filled cavity problem by assuming the gas undergoes isothermal compression. In 1933, Minnaert [2] considered bubble oscillations in a pool of liquid who succeeded in showing that the sound emitted by moving liquid arises from the radial pulsations of entrained bubble. The gas bubble in a viscous fluid under the assumption of isothermal conditions was studied by Barlow and Langiois [3]. Later, Langiois [4] studied the same problem by taking the effect of viscosity and diffusion. Yang and Yeh [5] performed a theoretical study of bubble behavior in Newtonian liquids. They analyze the bubble growth with time wise variations, collapse of bubble and energy dissipation for Newtonian and Non-Newtonian liquids. Amon and Denson [6] provide new idea that a spherical envelope of fluid is surrounded from a spherical bubble that supplies a limited amount of gas. The mathematical model for bubble growth in purely viscous liquid driven by gas diffusion was introduced by Arafmanesh and Advani [7]. Street [8] studied for the first time the effect of viscoelastics on bubble growth using the constitutive of an Oldroyd B fluid model. When compared to bubbles expanding in Newtonian liquids he found that bubbles expanding in a viscoelastic liquid with the same zero shear rate viscosity have a higher initial growth rate followed by a cessation of growth as bubbles interact to form thin liquid films. Like Street, Tanasawa and Yang [9] and Ting [10] studied the bubble increase in a same viscoelastic fluid.Han and Yoo [11] provide experimental results showing the effects of various parameters. Rasmusen and Campbell [12] studied bubble growth in a power law fluid and they compared own model results and the results derived from Arafmanesh and Advani [13] with experimental results. From there study, they concluded that the results obtained from an Oldroyd B model are good agreement with experimental results capturing both the rapid initial phase followed by a slower growth phase and the equilibrium bubble radius. The two viscous models show large deviations during the initial stages of growth and, since these models assume a limitless supply of gas, do not predict an equilibrium bubble radius. Koopmans et al [14] model foam growth in a thermoplastic polymer by analysing an isolated spherical bubble expanding in a generalized Newtonian fluid.

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As many applications of bubble dynamics are related to viscoelastic liquids, therefore we are interested to investigate the bubble dynamics with second grade viscoelastic liquids. According to the best of our knowledge, there is no study available in the literature about bubble behaviour obeys the constitutive equation of a second grade viscoelastic fluid.

2 Basic Equation

Basic equations governing the flow of an incompressible fluid are:

The equation of continuity

$$\nabla.\vec{u} = 0. \tag{1}$$

The equation of motion (with the neglect of external body forces such as gravity)

$$\rho \frac{D\,\vec{u}}{Dt} = \nabla \vec{T} + \rho \vec{f}, \tag{2}$$

where $\vec{u}$ is the velocity field, $\vec{T}$ is the Cauchy stress tensor, ρ is the constant density, $\vec{f}$ is the body force per unit volume $\frac{D\vec{u}}{Dt}$ is the material derivative and $\vec{T}$ is defined by $\vec{T} = -p\vec{I} + \vec{\tau}$.

The mass Transport equation is

$$\frac{\partial c}{\partial t} + \vec{u}\,.\nabla c = \nabla.(D\nabla c). \tag{3}$$

In which c is concentration and
$\nabla = e_r \frac{\partial}{\partial r} + e_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + e_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}.$

In spherical coordinates the governing Eqs. (1-3) are

Continuity equation:

$$\frac{\partial p}{\partial t} + \frac{1}{r^2}\frac{\partial(\rho r^2 u_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho u_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho u_\theta)}{\partial \phi} = 0. \tag{4}$$

r -component momentum equation:

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{r\sin\theta}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2 + u_\phi^2}{r}\right) =$$
$$\frac{\partial p}{\partial r} + \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\tau_{r\theta}\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right) + \rho g_r. \tag{5}$$

θ -component momentum equation:

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_\phi}{r\sin\theta}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} - \frac{\partial u_\phi^2 \cot\theta}{r}\right) = \frac{1}{r}\frac{\partial p}{\partial \theta} + \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right) +\right.$$
$$\left.\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\tau_{\theta\theta}\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\theta}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}\right) + \rho g_\theta. \tag{6}$$

ϕ -component momentum equation:

$$\rho\left(\frac{\partial u_\phi}{\partial t} + u_r\frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta} + \frac{\partial u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} + \frac{u_\theta u_\phi}{r}\cot\theta\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} + \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\phi}\right) +\right.$$
$$\left.\frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - 2\frac{\cot\theta}{r}\tau_{\theta\phi}\right) + \rho g_\phi. \tag{7}$$

Transport equation:

$$\frac{\partial c}{\partial t} + u_r\frac{\partial c}{\partial r} + \frac{u_\theta}{r}\frac{\partial c}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial c}{\partial \phi} = D\left[\frac{\partial^2 c}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 c}{\partial \theta^2} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 c}{\partial \phi^2} + \frac{2D}{r}\frac{\partial c}{\partial \phi}\right]. \tag{8}$$

For a second grade fluid the stress tensor $\vec{T}$ is defined as

$$\vec{T} = -p\vec{I} + \mu\vec{A}_1 + \alpha_1\vec{A}_2 + \alpha_2\vec{A}_1^2, \tag{9}$$

in which p is pressure, $\vec{I}$ is the identity tensor, μ is the coefficient of viscosity, α_1, α_2 are the normal stress moduli and $\vec{A}_1, \vec{A}_2$ are the line kinematic tensors defined by

$$\vec{A}_1 = \left(\nabla\vec{u}\right) + \left(\nabla\vec{u}\right)^T, \tag{10}$$

$$\vec{A}_2 = \frac{D\vec{A}_1}{Dt} + \vec{A}_1\left(\nabla\vec{u}\right) + \left(\nabla\vec{u}\right)^T\vec{A}_1. \tag{11}$$

The model given in Eq. (8) reported by Dunn and Fosdick [15], Fosdick and Rajagopol [16] is compatible if the material moduli must satisfy the following conditions: $\mu \ge 0, \alpha_1 \ge 0$ and $\alpha_1 + \alpha_2 = 0.$

3 Formulation of physical problem

A single bubble of an incompressible viscoelastic second grade fluid is considered. It is postulated that bubble shape may be expressed in spherical co-ordinate (r,θ,ϕ). The origin of co-ordinate system is fixed at the center of the bubble in which $R(t)$ is the distance from the center to the surface of the bubble, R_0 is the radius of the unperturbed bubble. The growth is effected by some main parameter such as difference between inside and outside presure Δp, non-Newtonian parameter $\alpha,$ shear stress τ and surface

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tension σ. The physical problem is described as shown in Fig. 1.

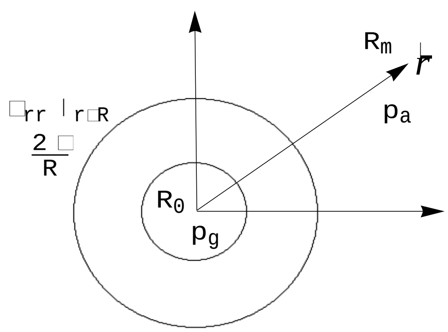

Figure 1: Geometry of Bubble Growth

The following assumptions are taking into account during analysis:

i. The shape of bubble is assumed to be spherical.
ii. The effects of gravity and gas pressure are negligible and the surface tension is kept constant.
iii. Further assuming that the gas is uniformly distributed inside the bubble.
iv. Pressure inside the bubble is assumed to be uniform.

The mathematical model describing the growth of a bubble consists of mass, momentum and diffusion equation.

Velocity field is

$$\vec{u} = \vec{u}\left(u_r(r,t), 0, 0\right), \ \tau = \tau(r,t). \tag{12}$$

Inserting the velocity field in equation (4), we obtain

$$\frac{\partial u_r}{\partial r} + \frac{2u_r}{r} = 0. \tag{13}$$

Integration of equation (13), under the assumption that the density ratio of gas inside the bubble to that of liquid is very small yields the following result

$$u_r = \frac{R' R^2}{r^2}. \tag{14}$$

Where R represent the bubble radius and R' is it's time derivative. Normal component of stress tensor after using velocity field becomes:

$$\tau_{rr} = -p + 2\mu\frac{\partial u_r}{\partial r} + \alpha_1\left(2\frac{\partial^2 u_r}{\partial t \partial r} + 2u_r\frac{\partial^2 u_r}{\partial r^2} + 4\left(\frac{\partial u_r}{\partial r}\right)^2\right) + 4\alpha_2\left(\frac{\partial u_r}{\partial r}\right)^2. \tag{15}$$

$$\tau_{\theta\theta} = -p + \mu\left(\frac{u_r}{r}\right) + \alpha_1\left(\frac{2}{r}\frac{\partial u_r}{\partial t} + 2u_r\left(\frac{r\frac{\partial u_r}{\partial r} - u_r}{r^2}\right) + 4\frac{u_r^2}{r^2}\right) + 4\alpha_2\left(\frac{u_r^2}{r^2}\right). \tag{16}$$

$$\tau_{\phi\phi} = -p + \mu\left(\frac{u_r}{r}\right) + \alpha_1\left(\frac{2}{r}\frac{\partial u_r}{\partial t} + 2u_r\left(\frac{r\frac{\partial u_r}{\partial r} - u_r}{r^2}\right) + 4\frac{u_r^2}{r^2}\right) + 4\alpha_2\left(\frac{u_r^2}{r^2}\right). \tag{17}$$

Using velocity field given in Eq. (10) and normal stress component in momentum Eq. (12-14) after integrating, we have

$$\frac{R}{r}\left(RR'' + 2R_2'\right) - \left(\frac{R'^2 R^4}{2r^4}\right) = \frac{p_g - p_a}{\rho} - \frac{6\alpha_1 R^4 R'^2}{\rho r^6}. \tag{18}$$

The dynamic equation of bubble growth may be obtain by replacing r by R as

$$RR'' + \frac{3}{2}R'^2 = \frac{p_g - p_a}{\rho} - \frac{6\alpha_1 R^4 R'^2}{\rho R^2}. \tag{19}$$

The substitution of equation Eq. (12) into Eq. (13), gives

$$\tau_{rr} = -p - \frac{4\mu R' R^2}{r^3} + \alpha_1\left(-\frac{4}{r^3}\left(R''R^2 + 2RR'^2\right) + 12RR'^2 + 12\frac{\left(R'R^2\right)^2}{r^6}\right). \tag{20}$$

The net force on the bubble in the radially outward direction per unit area is $\left(\tau_{rr}\right)_{r=R} + p_g = \frac{2\tau}{R}$ on substitution the value of τ_{rr} from Eq. (19), we have

$$p_a = p_g - \left(\frac{2\tau}{R} + \frac{4\mu R'}{R}\right) - \alpha_1\left(4\frac{R''}{R} - 4\frac{R'^2}{R^2}\right). \tag{21}$$

Combining Eq. (17) and Eq. (20), we obtain second order differential equation for bubble growth as

$$RR'' + \frac{3}{2}R'^2 + \frac{4\mu R'}{\rho R} + \frac{2\tau}{\rho R} = \frac{p_g - p_a}{\rho} - \frac{\alpha_1}{\rho}\left(\frac{4R''}{R} + \frac{2R'^2}{R^2}\right). \tag{22}$$

With initial conditions

$$R(0) = R_0 \text{ and } R'(0) = 0. \tag{23}$$

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Introducing the following non-dimensional quantities

$$R^* = \frac{R}{R_0},\ R'^* = R'\left(\frac{\rho m}{\Delta p}\right)^{\frac{1}{2}},\ R''^2 = \frac{R'' R_0 \rho_m}{\Delta p},$$

$$\mu^* = \left(\frac{\mu}{R_0}\right)\left(\frac{1}{\rho \Delta p}\right)^{\frac{1}{2}},\ p_a = p_a^* \Delta p + \rho_g.$$

In Eq. (20) and Eq. (21), after dropping the asterisks, we obtain the following set of equations:

$$RR'' + \frac{3}{2}R'^2 + \frac{4\mu R'}{R} + \frac{2\tau}{R} = p_g - p_a - \alpha_1\left(\frac{4R''}{R} + \frac{2R'^2}{R^2}\right). \tag{24}$$

$$R(0) = 1 \text{ and } R'(0) = 0. \tag{25}$$

To convert system of Eq. (23) and Eq. (24) into first order, we introduce

$$R = R_1$$
$$R' = R_2$$
$$R'' = R_2'$$

The system becomes

$$R_2 = R_1'. \tag{26}$$

$$R_1 R_2' + \frac{3}{2}R_2^2 + \frac{4\mu R_2}{R_1} + \frac{2\tau}{R_1} = p_g - p_a - \alpha_1\left(\frac{4R_2'}{R_1} + \frac{2R_2^2}{R_1^2}\right). \tag{27}$$

And the initial condition takes the form

$$R_1(0) = 1 \text{ and } R_2(0) = 0. \tag{28}$$

The solution of transport Eq. (8) is reported by [17] given as

$$C_r = C_R + \frac{1}{R}\left(\frac{S_0^3(C_0 - C_R) - \frac{\rho_g}{\rho_m}R^3}{\frac{S_0^3}{R} - \frac{3}{2}\left(S^2 - R^2\right)}\right)\left(\frac{1}{R} - \frac{1}{r}\right). \tag{29}$$

There is a lack of physical results about mass transport phenomena. Therefore we investigate graphically this phenomenon under the variation of different physical parameter.

4 Results and discussion

Bubble dynamics is investigated from equation (27-28) and (29). To understand the physics of the bubble growth, it is necessary to examine the effects of the concerned parameters of physical interest. In this perspective, a comprehensive analysis is made by plotting the dimensionless bubble radius and concentration for various values of the visco-elastic parameter, surface tension and for the pressure difference in Figs. (2-9). Parameters used to generate the figures are outlined in the figures captions though these are non-dimensional values. Figure 2 illustrates solutions for small to large visco-elastic parameter for a flow with pressure difference and surface tension is 0.2. Here, as expected, fluid of larger value of parameter will generate a small bubble in the same duration of time. The plots for bubble growth in Fig. 3 shows that for small values of surface tension the bubble grows from initial size and for large values of surface tension the bubble reduces from its original size. The reason is because surface force dominates other forces. The effects of pressure difference on the bubble radius are shown in Fig. 4. One can observe that as the pressure difference increases the bubble grows i.e. when the internal pressure inside the bubble become large as compared to the outside pressure on the surface of the bubble the bubble grows more rapidly and the process is reverse when the outside pressure is greater that the inside pressure. Radius variation versus time is plotted in Fig. 5 for various values of fluid viscosity. It is concluded that on reducing the fluid viscosity the bubble radius increases and this increase is seems to be more rapid for fluid having low viscosity. The Figs. 6 and 7 are plotted for rate of change of bubble growth for different values of the visco-elastic parameter and for the fluid viscosity respectively. Physically compatible results are noted here. On increasing the visco-elastic parameter and fluid viscosity the rate of change of radius increases. Mass transport phenomena are plotted in Figs. (8-9) for various values of initial concentration and density ratio and for parameter Variation of bubble mass concentration is shown in Fig. 8. Here, it is noticed that with increasing the initial concentration, the mass transport increases as the bubble grows. In fact, the rate of mass transfer with a large initial concentration is more than that of mass transfer in a bubble with a small initial concentration. The transport phenomena for different values density ratio are plotted in Fig. 9. Here, it can be seen that liquid which are dense will transport large concentration of liquid during growth process.

5 Conclusions

In this paper an attempt has been made to study the bubble dynamics for the flow of visco-elastic incompressible fluid. Bubble dynamics is modeled from governing continuity and momentum equation in spherical coordinate system in term of ordinary differential equation. The governing equation for bubble radius is solved using RK-4 method.

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During investigation of bubble dynamics the following conclusions are made:

(i) Larger value of visco-elastic parameter will generate a small bubble.

(ii) For small values of surface tension the bubble grows and the case is reverse for large values of surface tension.

(iii) For large pressure inside the bubble as compared to the outside pressure on the surface of the bubble the bubble grows more rapidly.

(iv) Mass transfer with a large initial concentration is more than that of mass transfer in a bubble with a small initial concentration.

(v) Fluids with high density will transport large concentration of liquid as bubble grows.

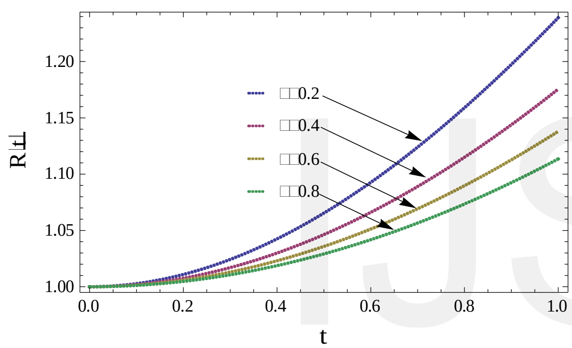

Figure 2: Bubble Radius versus time for various value of visco-elastic parameter α, for fixed values of $\mu = 0.1$, $p_g - p_a = 1$, and $\tau = 0$.

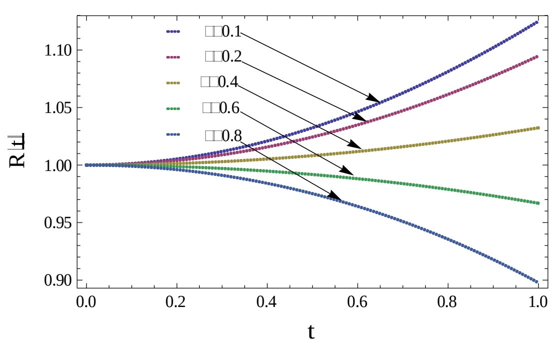

Figure 3: Bubble radius versus time for various value of surface tension τ, for fixed values of $\mu = 0.1$, $p_g - p_a = 1$, and $\alpha = 0.5$

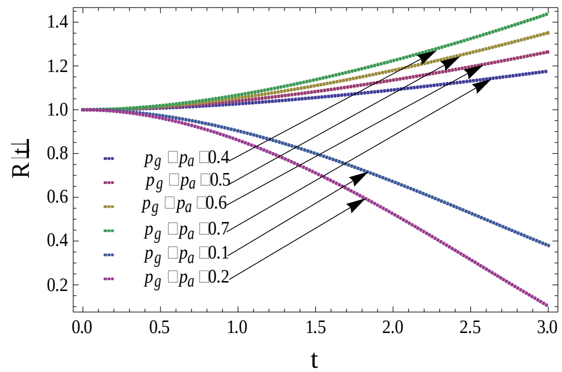

Figure 4: Bubble radius versus time for various value of pressure difference $p_g - p_a$, for fixed values of $\mu = 0.5$, $\alpha = 0.5$ and $\tau = 0.1$.

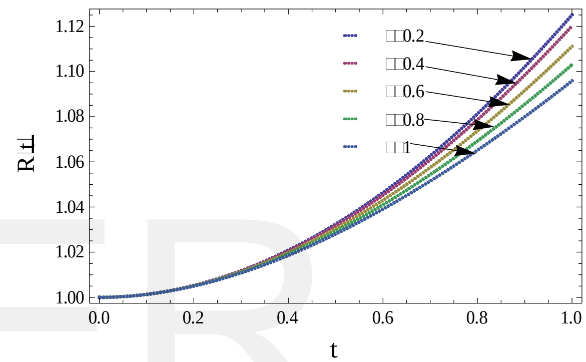

Figure 5: Bubble radius versus time for various value of fluid viscosity μ, for fixed values of $\tau = 0$, $p_g - p_a = 1$, and $\alpha = 0.5$

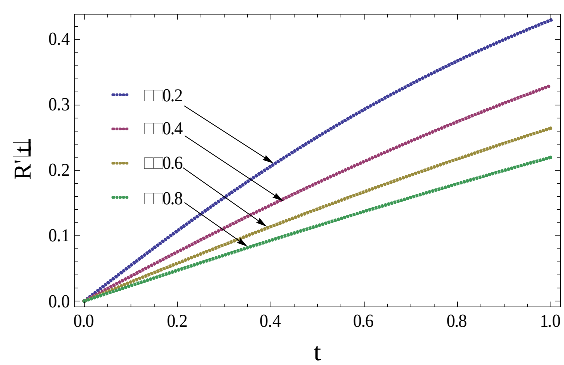

Figure 6: Rate of change of bubble radius for various value of visco-elastic parameter α, for fixed values of $\mu = 0.1$, $p_g - p_a = 1$, and $\tau = 0$.

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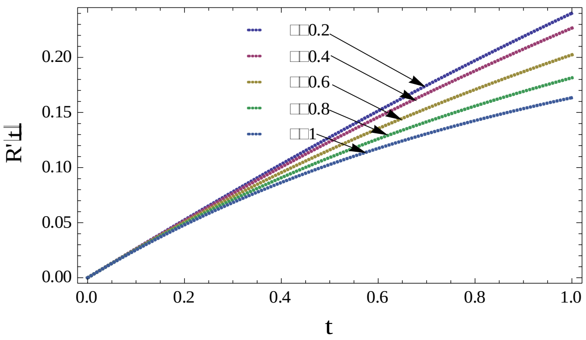

Figure 7: Rate of change bubble radius for various value of fluid viscosity μ, for fixed values of $\tau = 0$, $p_g - p_a = 1$, and $\alpha = 0.5$.

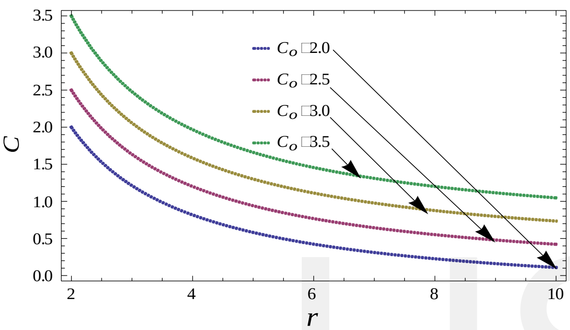

Figure 8: Mass transport profile for various values of initial concentration C_0, for fixed values of $p_g - p_a = 0.7$, $C_R = 1.5$, $R = 2$, $a = 2$, and $S = 1.8$.

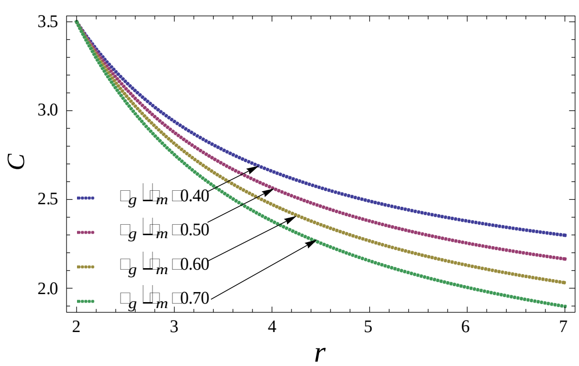

Figure 9: Mass transport profile for various values of density ρ_g / ρ_m, for fixed values of $C_0 = 3.5$, $C_R = 1.5$, $R = 2$, $a = 2$, and $S = 1.8$.

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